

STUDYING TORRES GUITARS

PART II: ACOUSTICS ON THE PATH TO THE MODERN GUITAR

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Abstract:

Antonio de Torres is celebrated to have introduced the shape and definition of the modern guitar that we find today. Andalusian and other guitar makers still continue to manufacture Torres models besides their own models of classical Spanish guitars and flamenco guitars. This paper seeks to document and explore some of the basic tuning concepts behind the investigated examples of Torres guitars, in the context of other epochs of guitar making, including modern guitars. For this purpose some 23 excellent master pieces from six different collections have been measured acoustically. The development of the guitars' sizes and shapes and their relation to basic acoustical properties is outlined, and further acoustical properties such as bass support, sound pressure, and timbre are investigated for the five Torres guitars under observation. The feature maps of these acoustical properties clearly support classification of different epochs of guitar making, one of which is the work of Torres.

Keywords:

classical Spanish guitar, flamenco guitar, Torres guitars, acoustics

ESTUDIANDO LAS GUITARRAS TORRES - PARTE II: ACÚSTICA EN EL CAMINO A LA GUITARRA MODERNA

Resumen:

Antonio de Torres es conocido por haber creado la forma y la definición de la guitarra moderna que encontramos hoy. Los fabricantes de guitarras andaluces junto con otros siguen fabricando modelos de Torres hoy en día además de sus propios modelos de guitarras clásicas españolas y guitarras flamencas. Este ensayo busca documentar y explorar algunos de los conceptos básicos de afinación en base a ejemplos de investigación de las guitarras de Torres, en un contexto de otras épocas de fabricación de guitarras, incluyendo guitarras modernas. Para este propósito, se han medido acústicamente unas 23 excelentes obras maestras de seis colecciones diferentes. El desarrollo de los tamaños y formas de las guitarras en relación a las propiedades acústicas básicas son descritas junto con una investigación de otras propiedades acústicas como: el soporte de bajos, la presión del sonido y el timbre para cinco guitarras Torres. Los mapas de características de estas

propiedades acústicas reflejan claramente la clasificación de diferentes épocas de fabricación de guitarras, una de las cuales es el trabajo de Torres.

Palabras clave:

guitarras clásicas españolas, guitarras flamencas, guitarras de Torres, acústica

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INTRODUCTION

Antonio de Torres (* 13. June 1817, † 19th Nov. 1892) is said to be one of the greatest guitar makers ever. From his first and second period of crafting guitars, some 65 are documented in (Romanillos, 1997) by pictures, by some measures, and by description of some construction specials. Torres is referred to as having introduced the shape and definition of the modern guitar that we find today. Romanillos reports about Torres' dialog with musicians and how he strived for a guitar with volume and projection for solo concerts. This is a considerable target since the guitars in those days were used for accompanying chanting or dancing and had little attention as solo instrument.

While considering guitar construction innovations of his time, experimenting, and compiling constructive features, Torres' important contribution to the modern guitar is the size of the plantilla. Compared to guitars of his time the size of the plantilla increased significantly from an average of 1085 cm² for the romantic guitar (Dausend, 2002) to an average of 1235 cm² for the guitars documented from Torres' first and second epoch (Romanillos, 1997). However, Torres was searching for perfect proportions as can be read from the wide variety of sizes and shapes. Many of today's guitar makers, who produce classical and flamenco guitars, also offer copies of Torres guitars. In Granada alone there are more than ten guitar makers offering such copies. However, when asking them about which of the many sizes and shapes they refer to, there are no consistent answers.

PLANTILLA SIZE AND FREQUENCY OF THE HELMHOLTZ CABINET

When comparing the plantilla sizes of the past, the examples of Torres, and the present, the general evolution becomes clearer. Fig. 1 documents the plantilla size versus the fundamental frequency of the Helmholtz cabinet, which will be denoted by f_0 in this paper. The plantilla size is estimated by Romanillos' formula (1997)

$$A_{plantilla} = \frac{L \cdot (w_u + w_w + w_l)}{3} \cdot 0.96 \quad (1)$$

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Basically the length of the top plate, L , is multiplied by the average of the upper and lower width, and the width at the waistline, and by a correction factor.

The frequency f_0 is calculated by geometry (Meyer, 1985, p. 31):

$$f_0 = \frac{c}{2\pi} \cdot \sqrt{\frac{\pi \cdot R^2}{V \cdot \left(h + \frac{\pi}{2} \cdot R\right)}} \quad (2)$$

where c is the sound velocity, h is the plate strength at the hole, V is the air volume in the cabinet, and R is the radius of the hole. Here, for volume V , the plantilla area is multiplied by the average of the rib height minus a minor correction factor assuming that the arching at top and back side already compensate for a lot of the bracing volume inside. Therefore f_0 reflects more or less the volume of the cabinet in relation to the hole size, and therefore most of the f_0 variation for plantillas of comparable size comes from rib height. The frequency for the air mode as finally measured on the various instruments is usually lower than f_0 , and the reason for this will be discussed later.

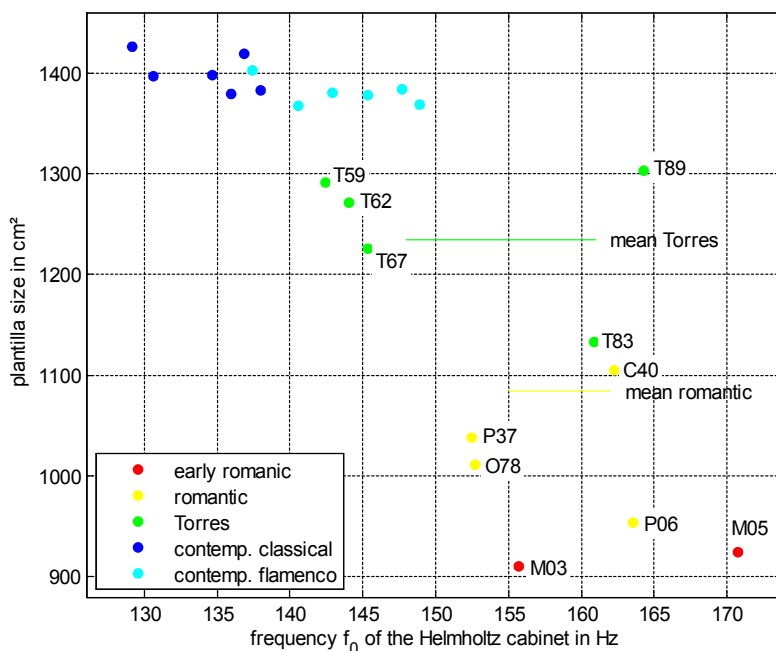


Fig. 1. Plantilla size versus fundamental Helmholtz cabinet frequency f_0 across various periods of guitar making. For acronyms see Table 1, for non-labelled examples of guitars see text.

The guitars considered in Fig. 1 are first of all the five Torres guitars, but also four romantic guitars, and two 12-string early romantic guitars. All guitars are in good condition without cracks or holes and described in part I of this publication (Avalle and González, 2019). For representation of contemporary guitars there are three flamenco and three classical guitars from Granadian guitar makers, all built in 2018. And there are three flamenco and three classical guitars from the years 1971 to 1981 from a vintage collection in Hamburg (2 x Granada, 1 x Madrid in each set of 3 guitars). These guitars are acoustically documented in (Mores, 2019).

<i>acr.</i>	<i>year</i>	<i>maker</i>	<i>presently hosted</i>	<i>remarks</i>
M03	1803	Manuel Munoa	collection Romanillos	early romantic guitar, 12
M05	1805	Francisco	collection Romanillos	early romantic guitar, 12
P06	1806	José Pagés	Museo Barcelona #452	romantic
P37	1837	Francisco Pagés	Museo Barcelona	romantic, no reference
C40	1840	Benito Campo	collect. Daniel Gil de	romantic
T59	1859	Antonio de Torres	Museo Barcelona #626	
T62	1862	Antonio de Torres	Museo Barcelona #625	pappmache ribs and back
T67	1867	Antonio de Torres	collect. Daniel Gil de	no. 32, first epoche
O78	1878	Francisco Ortega	collect. Daniel Gil de	romantic
T83	1883	Antonio de Torres	Centro Doc. Mus.	no. 46, second epoch
T89	1889	Antonio de Torres	Museo Barcelona #12107	lady size guitar

Table 1. Reference guitars under investigation sorted by year of production, among others (see text).

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Fig. 2. Pictures of guitars made by Antonio de Torres in the years 1859, 1962, and 1889, from left to right.



Fig. 3. Pictures of guitars made by Antonio de Torres in the years 1883, and 1867, from left to right.

Unfortunately, the admired “La Leona” of Torres is not included in the data set, as it was not possible to contact the owner.

Clearly, Torres has increased the plantilla size significantly compared to romantic and earlier guitars. Contemporary guitars, 100 to 150 years after the days of Torres, have an even larger. There is no significant difference between guitars from the 70/80ties and from 2018, and no significant difference between flamenco and classical guitars. Size development seems to have saturated at about 1400 cm², with little variation, whereas Torres’ guitars show a wide variation, expressing the experimental character of his work.

Over the same period of evolution of guitars, the Helmholtz cabinet frequency f_0 decreased significantly, from 152 ..171 Hz for romantic guitars, to 142 ... 164 Hz for Torres guitars, to 137 .. 149 Hz for contemporary flamenco guitars, and 129 ..138 Hz for contemporary classical guitars. This is important as the guitars makers sought to support resonance and radiation for the very low notes, remembering that the low E-string is tuned to 82.4 Hz. Now, it is not surprising that the larger guitars come with a lower f_0 since volume is the main parameter in Eq. 2. However, note that for contemporary guitars there is little variation in plantilla size, and a large variation in f_0 . This comes mainly from variation of rib height (0.83 Hz/mm in this data set) and hole diameter (1.71 Hz/mm).

FREQUENCY DIVERGENCE FOR COUPLED MODES

Before exploring the acoustical characteristics of Torres and other guitars it is useful to recall some fundamentals on vibrational acoustics. First, there are several main resonating components at a guitar: top, air enclosed in the cabinet, and back. Of course the neck is also vibrating at some 40 to 80 Hz, however, with little potential for excitation and radiation. And there are many other vibrational patterns of higher frequency possible at each plate. Presently, we focus on the so called signature modes (Gough, 2013). These are the modes which are well determined by geometry, material parameters and manufacturing, while higher modes are poorly controllable and should be treated statistically. Second, resonating components in the same system may couple with each other, effectively resulting in divergent frequencies. This means that frequencies for the coupled resonating system components diverge from the nominal frequencies of the individual components when operated stand-alone.

Warburton (1954) and Caldersmith (1984) have investigated this analytically for plates, and the Fletcher and Rossing (1998) compilation suggests various equivalent electrical resonators for modelling coupled mechanical resonators. For parallel circuits, they determine the frequency divergence by

$$(f^2 - f_a^2) \cdot (f^2 - f_b^2) = f^4 \cdot k^2 \quad (3)$$

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Equation (4.17) in (Fletcher and Rossing, 1998). f_1 and f_2 will result from solving the quadratic equation for f . The divergence increases with increasing coupling factor k and the closer the frequencies of the individual components approach each other. This is visualized in Fig. 4. According to Fletcher and Rossing (1998), a system consisting of top plate, air, and back plate, all mutually coupled with each other, rather follows a “serial” paradigm. The analogue divergence curve is given in Fig. 4 by dotted lines. Applying this model to the empirical data of Meyer fits well. His regressions on the experimental results from 50 differently tuned top plates coupled with an air cavity follow the dotted lines. The regressions return normalized frequencies 1.25 and 0.74 at $f_a = f_b$, while the model predicts 1.225 and 0.707 (factors are indicated in Figure 2). Meyer determined a coupling factor $k = 0.5$ from his experiments¹.

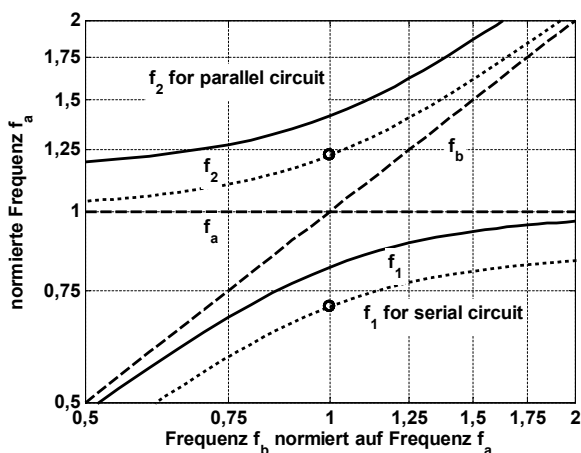


Fig. 4. Normalized frequencies f_1 and f_2 resulting from coupling two system components with individual stand-alone frequencies f_a and f_b , when coupled by a factor of $k = 0.5$.

Applying these findings to the guitar results in a considerable frequency divergence. For example, a typical top plate with its strong fundamental mode (0,0) at $f_{top} = f_a = 160$ Hz is coupled with a typical fundamental air cavity resonating at $f_0 = f_b = 135$ Hz. Normalized $f_a = 1$ and $f_b = 0.844$. The resulting frequencies are $f_1 = 6.64$ and $f_2 = 1.14$, normalized, and 103 Hz and 183 Hz applied to the guitar. This means that the Helmholtz-resonance will not be found at 135 Hz but at 103 Hz. Likewise the (0,0) mode of the top is shifted up to 183 Hz. Meyer explains the effect of lowering the air resonance by the fact that the given softness of

¹ For an estimate of confidence in Meyer’s determination, a factor of $k = 0.48$ (0.52) returns 1.217/0.721 (1.233/0.693) from prediction.

the top compared to a rigid plate will soften the spring character of the compressed / decompressed air inside the cabinet, effectively decreasing the resonance frequency. Likewise, the more or less stiff top plate becomes somewhat stiffer when resting on a layer of air that must be compressed before giving up its thickness, effectively increasing the resonance frequency.

This simple model will not yet allow to predict the resonances of the fully assembled guitar. There is also the back plate and the neck as additional components. Meyer extended the his model to include the back plate as this is also coupled with the enclosed air. There is additional an additional path of coupling via the ribs, and this becomes complex because of the several degrees of freedom at component transitions.

However, this brief excursion outlines the complex tuning problem a luthier is facing. It is one task to design the geometry of a “box” to deliver a specific frequency f_0 , and another task to tune the finished but not yet assembled top plate to a specific knocking tone. But it is yet another task or challenge to predict the frequency divergence for the coupled modes. This is true for two system components, and a real challenge for the given complexity of a guitar with more than two components and unknown coupling factors. This might be kept in mind while visiting Torres and his target to support the bass fundament on his guitars.

MEASUREMENT SETUP

Mobility Y expresses the willingness of a structure to move when exposed to force, $Y = v / F$ in [m/Ns]. Y is frequency dependent and reveals structure borne resonances. Mobility is measured at the guitar’s bridge to represent the case of plugged strings applying dynamic forces to the bridge. Mobility measured at the top is a valid representation since the top is the main radiating component. With just one point of measurement already most of the vibrational patterns, or modes, of the guitar can be captured. Here, a second point of measurement is added to achieve a somewhat improved holistic view, and yet another point of measurement is introduced in a symmetric arrangement to check for symmetry of guitar design. The total arrangement is shown in Fig. 5. Acceleration sensors (Kistler Type 8778A500, 0.4 grams) are placed at the bass side and the treble side of the bridge, see red marks in Fig. 5. The bridge is driven with an impulse hammer (DYTRAN 5800SL, 9.8 grams) between pairs of string attachment locations, on top of the bone inlay, see blue marks. Radiation is measured by pressure microphones located at the far end of the bridge, see green marks, but 10 ± 0.1 cm above the top. For structural analyses, the matrix comprises 3 inputs x 2 outputs = 6 mobility measurements, two of which are considered in this paper, the mobility at the bass side and at the treble side. For radiation analyses also only two from 6 optional cases are considered in this report.

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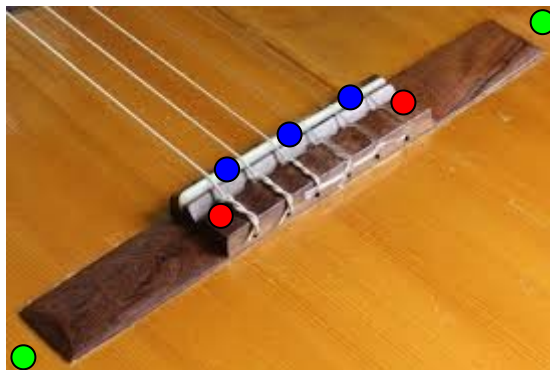


Fig. 5. Arrangement for structural and radiation measurements on guitars, with locations of introducing mechanical impulses with a light hammer (blue), locations of acceleration measurement (red) and locations of sound pressure measurement 10 cm above green marks.

Measurements were taken in the different places of the collections and not in a free-field. To avoid signal impairment by the characteristics of the given semi-reverberant rooms, an absorber was placed underneath the guitars, on the floor. The absorber was specifically designed for mobile measurements and for reaching down to 100 Hz (Mores, 2018). Measurement setups were placed in the center of respective rooms, with usually more than 3 m distance to the walls, resulting in some 20 ms reflection-free impulse responses for radiation measurement.

MEASURED MOBILITY

The mobility plot reveals a deterministic region below 400 Hz, and a statistical region above 400 Hz, see Figs. 6 and 7. This can be well observed by the overlay of mobility plots from twelve contemporary instruments, see Figure 7, graph in the bottom right. These two regions are also observed in other instruments, such as the violin with a turning point at 700 Hz (Bissinger and Mores, 2015), and the grand piano with a turning point at 200 Hz (Bertaud et al., 2003). In the deterministic region the air resonance at 100 Hz, the fundamental top mode, or (0,0) mode, at 200 Hz and the first cross mode, or (0,1) mode, at 230 to 300 Hz have been reported by many researchers, already (Torres and Boullosa, 2009) (Firth, 1977) (Elie et al., 2012) (Christensen and Vistisen, 1980). The deterministic region is also referred to as signature modes, since most of the properties can be controlled by the luthier and are therefore subject to sound tuning.

Most guitars have a fundamental air mode, A_0 , at around 100 Hz, and a fundamental top mode, the (0,0) mode, at around 200 Hz, see Figure 6, graph in the center left. A_0 is lower than f_0 according to the arguments outlined before. Likewise, the (0,0) mode of the top is shifted up in frequency due to coupling, compared to a situation when the top is only framed but not coupled to an air cabinet. For comparison, f_0 is indicated, (o) in the graphs of Figs. 6 and 7.

Next to the (0,0) mode, there is the cross mode, or (0,1) mode, indicated in Figure 6, graph at the bottom left. This mode represents a rotating bridge, with the rotation axis parallel to strings. This mode is well developed for P37, C40, M03, and M05, see Figure 6, and for Torres' guitars made in 1859, 1862, and 1867, and for all contemporary guitars measured, see Figure 7.

Next to A_0 , towards the lower end, there is another usually weaker peak for most of the guitars. The two optional explanations are: (i) the back plate is tuned to a very low fundamental frequency, effectively splitting up the air resonance into two resonances, according to the principles outlined above, or (ii) the resonance of the neck, which typically ranges from 40 Hz to 80 Hz, is very high.

There is a rather clear concept of tuning for the romantic guitars. A_0 is located at around 100 Hz to 110 Hz (all but M05), and the (0,0) mode above 210 Hz (all but P06). Torres followed another principle of tuning. While targeting for sound volume in larger presentation spaces, he also intended to support the very low frequency. His five guitars under observation revealed an A_0 below 92 Hz, well apart from the romantic guitars. Two guitars even reach down to 76 Hz (T62) and 73 Hz (T59), which is even well below the frequency of an open E string (82.4 Hz). While aiming for low frequencies, the resonances below 100 Hz peak in a single frequency. Only T67, which seems to follow a tuning principle like the C40 or M03, has two strong and distinct peaks. These cannot really be explained by the back plate or the neck. However, Torres' other guitars have a strong bass support. The support comes not alone from the low frequency, but also from the significant high level of mobility, up to ten times larger for Torres guitars than for romantic guitars.

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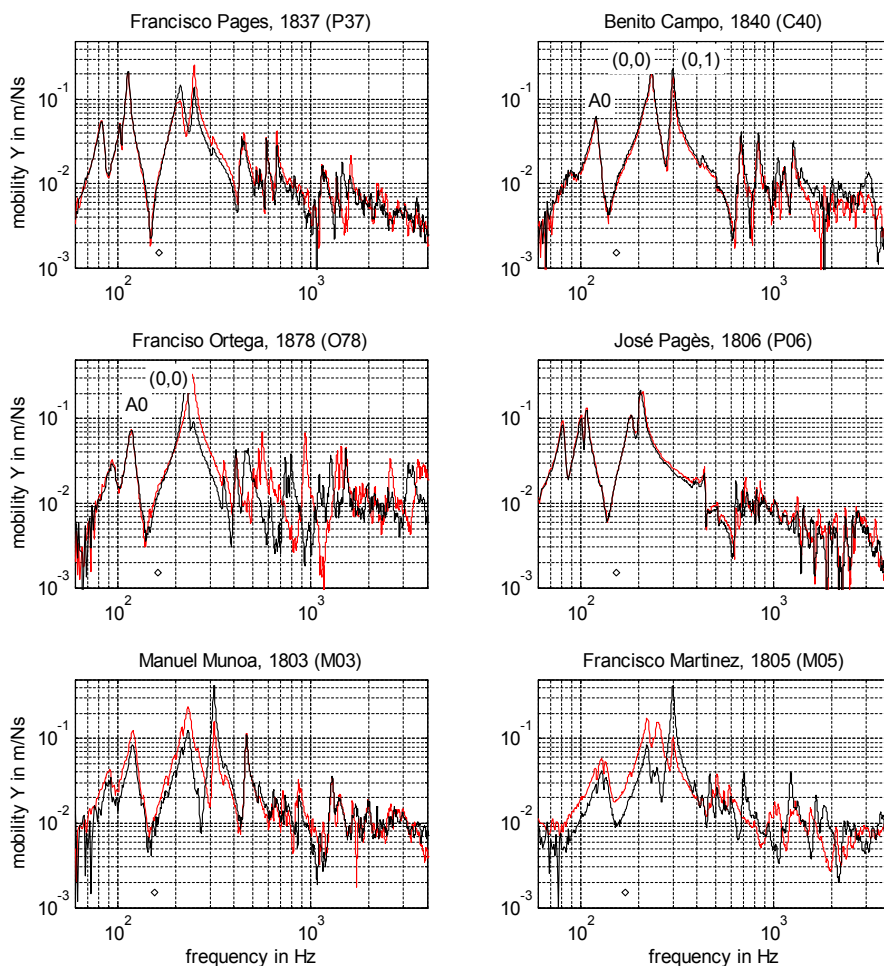


Fig. 6. Mobility plots of romantic guitars P37, C40, O78, P06, and the early romantic guitars M03 and M05. Mobility measured according to Fig. 5 on the treble side (red) and the bass side (black) of the bridge. Resonance of the Helmholtz cabinet f_0 as determined by volume alone is indicated by black circles.

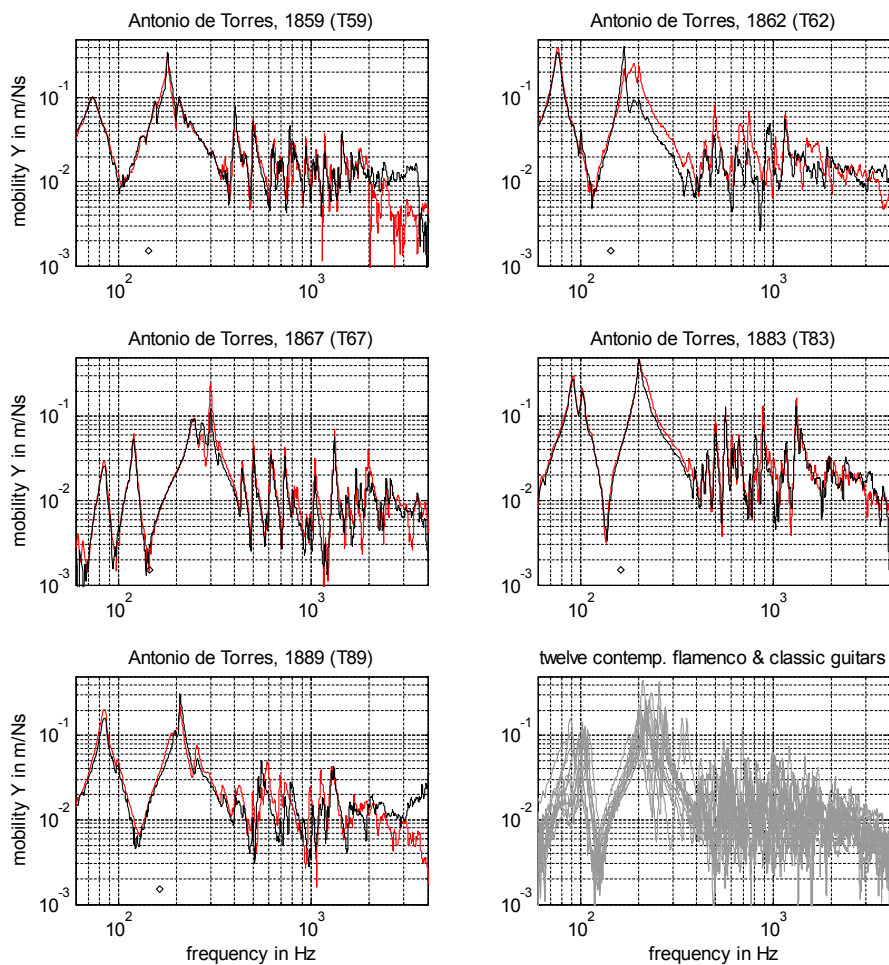


Fig. 7. Mobility plots of Torres guitars T59, T62, T67, T83, T89, and of twelve contemporary classic and flamenco guitars. Mobility measured according to Fig. 5 on the treble side (red) and the bass side (black) of the bridge. Resonance of the Helmholtz cabinet f_0 as determined by volume alone is indicated by black circles.

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Figure 8 summarizes some of the tuning principles that can be read from A_0 and the (0,0) mode. In general, for all guitars, there are some obvious trends. Fig.1 already indicated the trend of a gradual lowering of f_0 that came along with the growing plantilla. The same trend can be read from the gradual lowering of the frequency of A_0 , see the romantic guitars and early romantic guitars on the right hand side of Fig. 8, and Torres and contemporary guitars on the left side. However, Torres guitars were tuned lower than contemporary guitars are today. This seems to be contradictory: contemporary classical guitars have a 10 Hz to 20 Hz lower f_0 than Torres guitars as their plantilla is 100 cm² to 150 cm² larger than Torres' plantillas, but the frequency of A_0 in the classical guitars is roughly 20 Hz higher than in Torres guitars. This can be explained by the thin top plates in Torres guitars, the flexibility of which shift the frequency of A_0 down. Probably, a likewise soft back plate increases the effect of frequency shift, but this has not been checked during measurements.

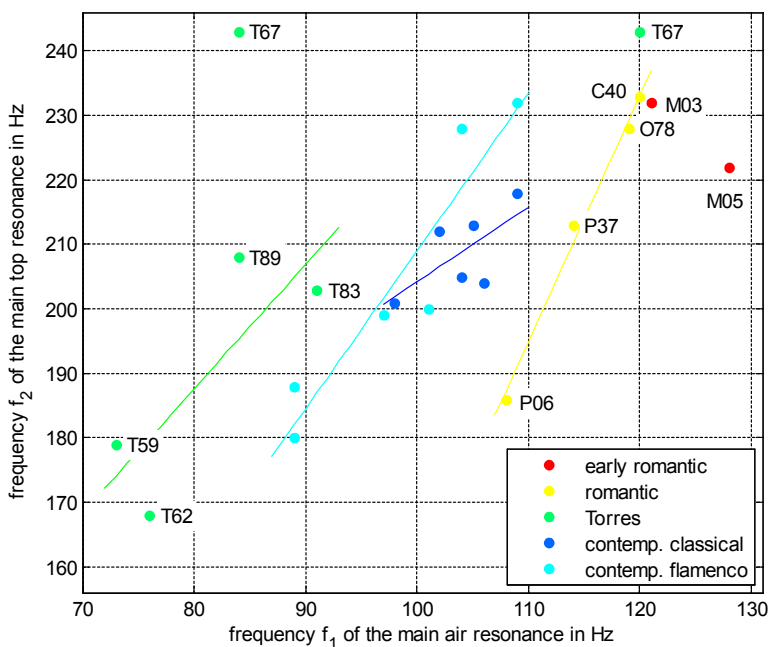


Fig 8. Frequency of the main top resonance, the (0,0) mode when coupled to the air cabinet in the assembled guitar versus the frequency of the main air resonance, for various epochs of guitar making. For acronyms see Table 1, for non-labelled examples of guitars see text.

The question raises why contemporary guitars ended up with the large plantilla if it was not for the support of low frequencies for A_0 ? Obviously, makers of contemporary guitars are

happy to support the bass with A_0 at 90 Hz to 100 Hz (flamenco) and 100 Hz to 110 Hz (classical) while the support for the open E string comes from the additional minor peak, a neck resonance or something similar. So it seems, from today's perspective, that Torres overdid the tuning to the low end.

The other tuning trend is that lowering of the A_0 frequency walks hand in and with lowering the frequency for the (0,0) mode. All regressions for each of the epochs show this trend, see Fig. 8. This relation comes again from the mutual coupling between the top and the air enclosed in the cabinet. Interestingly, the epochs can be clearly separated in the population map. This observation promotes the argument that the tuning of A_0 and the (0,0) mode are an issue on its own during the handcrafting and listening process, and that the shifts from epoch to epoch are another issue on its own, resulting from the other parameters of construction and tuning: plantilla size, hole size, rib height, and plate stiffness.

The experimental character of Torres' work can be read in the figure in several ways. The strong shift of Torres' trend line against the trend line of his predecessors. Secondly, the spread of observations is wide for Torres and narrow for romantic and temporary guitars. The good fitting for romantic guitars ($R^2 = 0.992$) and for contemporary flamenco guitars ($R^2 = 0.882$) encourages the argument, that the tuning has been and is well established in respective epochs, while Torres ($R^2 = 0.502$) was searching for creative options. Please note that T67 was not included in the regression because it is not clear which of the two strong peaks at 85 Hz and 125 Hz is the air mode, therefore both strong peaks are included in the population. T67 supports the argument of experiments in Torres' workshop, again. Surprisingly, the spread within the work of Torres is larger than the spread across different guitar makers within both epochs, romantic and modern.

MEASURED RADIATION

Radiation is measured with sound pressure microphones in the near-field, 10 cm above either end of the bridge, see Fig. 5. Near field is preferred against the far-field to avoid room characteristics impairing the responses from guitars. All responses in Figures 9 and 10 are normalized to 1 Newton hammer force, while the force of applied strokes was typically between 10 and 20 Newton.

The observations are:

- (i) The radiation response reflects the characteristics of the individual structural responses. This is to be expected, as mobility translates to radiation.
- (ii) The strong drops appearing in the structural responses, for instance close to f_0 for all guitars as marked in Figures 6 and 7, are not that strong in the radiation. This observation is the same as for responses from violins or from pianos. One explanation is that the impulse already causes radiation while travelling up and down a sound board at least once before establishing a mode.

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(iii) Another observation is, that all guitars radiate at quite similar levels in the high frequency range above 1 kHz. Only the contemporary guitars seem to radiate somewhat stronger in this range compared to the Torres and the romantic guitars, and the early romantic guitars under observation.

(iv) A very important observation is that the guitars from Torres radiate very weakly at low frequencies, with the exception of T67. All romantic guitars and early romantic guitars reach levels between 2 and 3 Pa/N somewhere in the range of 100 to 120 Hz, with the only one exception of P06 reaching lower levels. The guitars from Torres, however, reach only lower levels: clearly less than 1 Pa/N for T59, T62, and T89, and 2 Pa/N for T83. The exception is T67 with 4 Pa/N at 120 Hz. The twelve examples of contemporary guitars reach levels between 2 and 8 Pa/N, most of them well above the romantic guitars, and very well above the guitars from Torres. To summarize the somewhat curious observation: even while the tuning of romantic guitars C40, P37, and O78 does not reach down as far as it does for the guitars from Torres, their bass fundament in radiated sound is stronger. Or, in other words, Torres' obvious objection to support low frequency response by tuning down to low frequencies does not translate to radiation of such low frequencies.

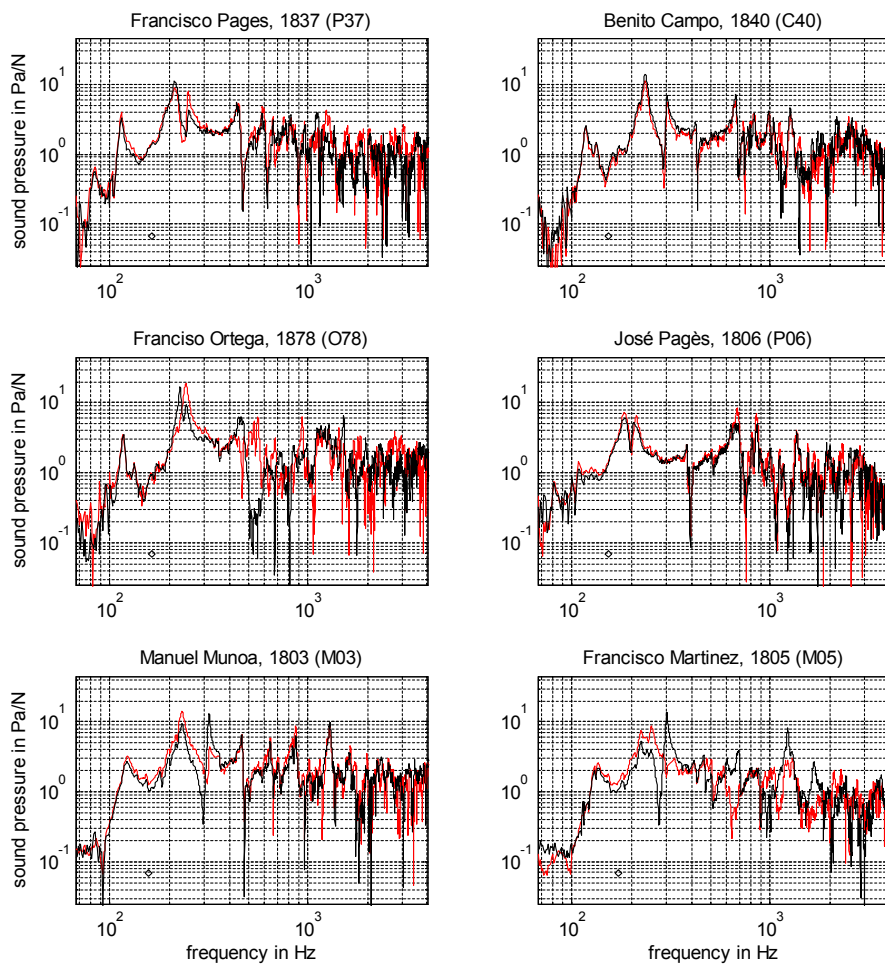


Fig. 9. Radiation plots of romantic guitars P37, C40, O78, P06, and early romantic guitars M03 and M05. Radiation measured according to Fig. 5 on the treble side (red) and the bass side (black) of the bridge. Resonance of the Helmholtz cabinet f_0 as determined by volume alone is indicated by black circles.

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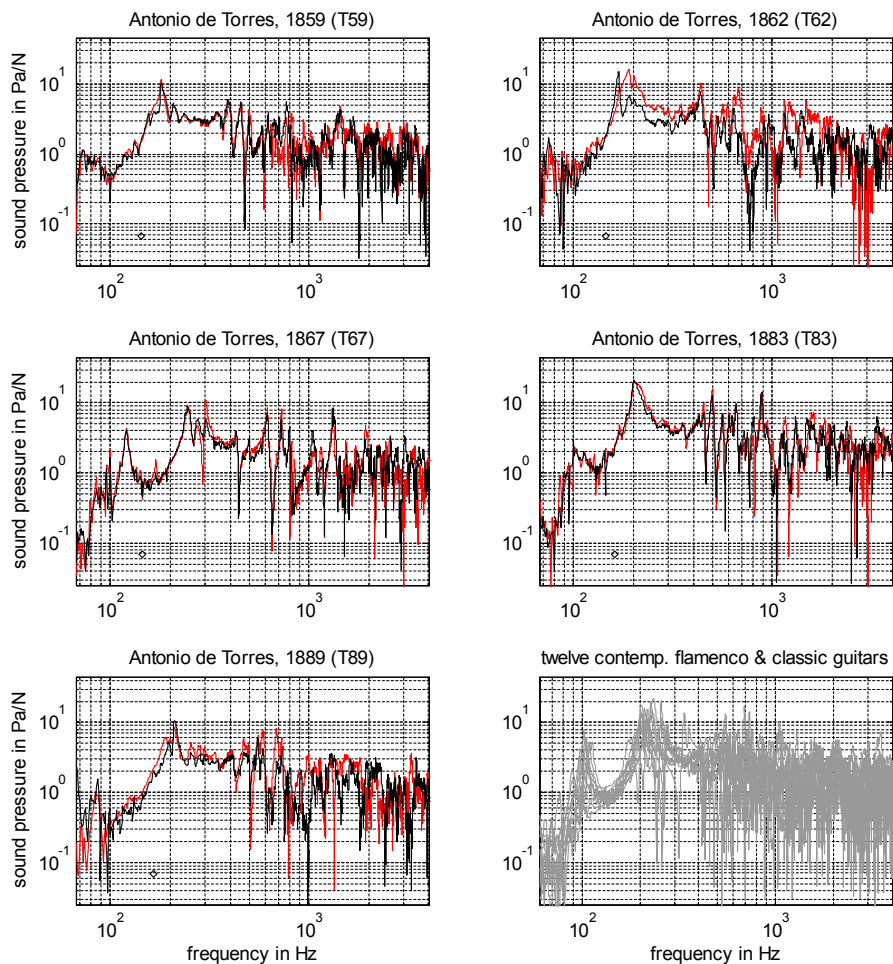
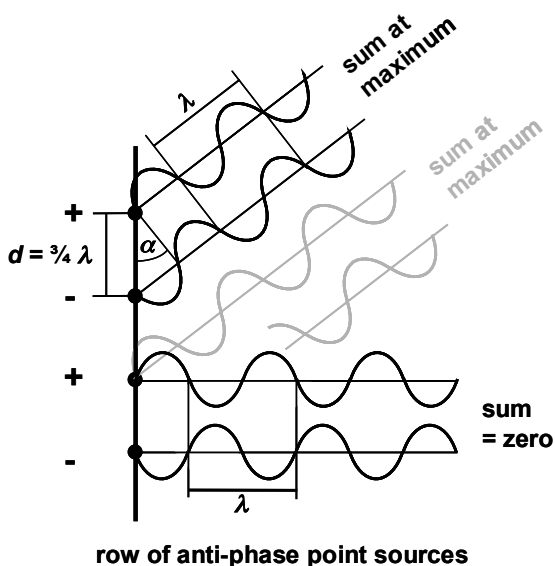
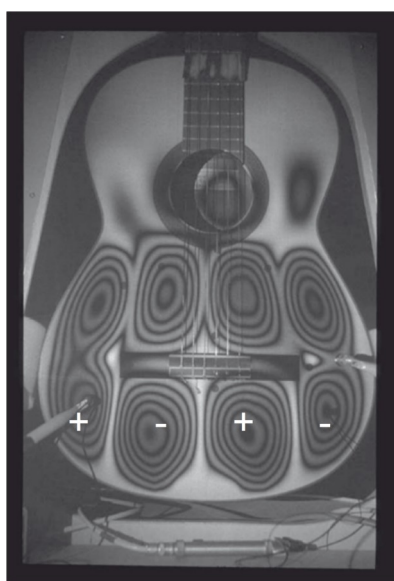


Fig. 10. Radiation plots of Torres guitars T59, T62, T67, T83, T89, and of twelve contemporary classic and flamenco guitars. Radiation measured according to Fig. 5 on the treble side (red) and the bass side (black) of the bridge. Resonance of the Helmholtz cabinet f_0 as determined by volume alone is indicated by black circles.

The explanation for this curious observation comes from plate thickness h . The most well-known observation is that knocking tones on thick plates pitch higher than on thin plates,

while leaving the other dimensions unchanged. This is reported in numerous publications and the analytical approach is summarized by Fletcher and Rossing (1989). The bending mode frequency f_{bend} grows directly proportional with plate strength, $f_{bend} \propto h$. Contrary to this, the so-called critical frequency f_{crit} grows inversely proportional with plate strength, $f_{crit} \propto 1/h$. The critical frequency is the frequency above which the radiation becomes efficient. Simultaneous measurement of radiation R and mobility Y lead to $R_{eff} \propto f^2$ for frequencies below f_{crit} and $R = 1$ for frequencies above f_{crit} , as observed in violin acoustics (Bissinger and Mores, 2015).

The logic behind the critical frequency is that the geometry of adjacent opposite-phase antinodes on a plate suggests $d > \lambda/2$ for effective radiation, with distance d between antinodes (material properties define the speed of sound and the established modes) and the wavelength λ for a specific frequency in air. Figure 11 illustrates this problem of geometry for the emerging sound wave. The distance d between antinodes must be larger than $\lambda/2$ before the beam is lifted from the plane of the top². The larger the sound velocity in the plate, the larger the distance d between antinodes compared to the wavelength λ (in air), and the lower the critical frequency, at which radiation becomes efficient. Radiation efficiency and projection is the true reason for the luthiers to use fast wood.



² It is only with high frequencies that the critical value of $d = \lambda/2$ can be reached because the distance between antinodes grows with the inverse of the root of frequency, while the wavelength in air grows with the inverse of frequency.

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Figure 11. Left: hologram of a guitar top resonating at a (4,2) mode (Richardson and Roberts, 1983), right: problem of geometry for the emerging sound wave, the distance d between antinodes must be larger than $\lambda/2$ before the beam is lifted from the plane of the top, $d > \lambda/2$.

In summary, the target is a low critical frequency which can be achieved with fast wood and with thick rather than thin plates. Why thick plates? Now, the frequency of a (m,n) mode is determined by plate strength h , by longitudinal and radial velocity, c_l and c_r , and by plate dimensions L_l and L_r .

$$f_{mn} = 0.453 \cdot h \cdot \left[c_l \cdot \left(\frac{m+1}{L_l} \right)^2 + c_r \cdot \left(\frac{n+1}{L_r} \right)^2 \right] \quad (4)$$

This means that one and the same frequency can be obtained for fast wood only in larger plates, the distance between antinodes grows with velocity for a given frequency. And h and c work in the same direction in above formula. So f_{mn} raises with h , and f_{crit} falls with h .

Now the target is a low critical frequency. Violin acoustics report $f_{crit} = 4 \dots 5$ kHz along the grain and $f_{crit} = 7 \dots 14$ kHz cross grain (Cremer, 1984). That means that radiation is most efficient above 4 kHz. Plates are usually of same or somewhat less strength in guitars, so the same or a somewhat higher value is assumed here for the moment.

Coming back to the guitars of Torres, this radiation efficiency is the most obvious reason why the very low tuning is probably more a problem than a solution. Why? Torres' tuning of A_0 down to values as low as 74 Hz can only be achieved by very soft tops and backs, compare Figures 1 and 8 for the discrepancy between f_0 of the Helmholtz frequency of the rigid-body air cabinet (Fig. 1) and the finally achieved air resonance A_0 in the assembled guitar (Fig. 8). In comparison with contemporary classical guitars, f_0 in Torres' guitars is typically 10 Hz higher, but A_0 is 20 to 30 Hz lower. This can only be achieved with very flexible plates. These flexible and therefore much thinner plates, compared to contemporary guitars, radiate therefore less efficiently.

In the end of the 19th century the war on loudness influenced the construction not only of pianos and of bowed instruments, but also of guitars. Loudness became a key feature for the otherwise unamplified musical instruments. This is likely to be the reason why guitar makers after the times of Torres started to optimize radiation by maintaining a certain minimum strength for the plates while tuning the air resonance A_0 down by means of an even larger plantilla, rather than by flexible plates. While A_0 does not reach down far enough to directly support the open E string at 82.4 Hz, other resonances such as the one coming from the neck is used to compensate for that little gap at the lowest end. This can be seen already on the romantic guitar P37, Fig. 9. Francisco Pages obviously understood to solve the compromise between low tuning and radiation in the same way as contemporary guitar makers do (Fig. 10 graph at the bottom right). Among the observed instruments from

Torres, however, there is the guitar T67 which reveals a lot of what contemporary guitars target at (Fig. 10).

Timbre

Another perspective on the guitar under observation is developed, when plotting the spectral centroid versus the normalized total radiated power, Fig. 12. The spectral centroid is calculated by the MIR toolbox, using a down-sampling rate of 12 kHz and a half cosine window across the 100 ms windows of sound which contain the impulse response taken with the microphones. The spectral centroid represents the timbre, and even though the outcome of such analysis depends on the sampling rate, the classification result is very stable across a wide range of sampling rates. For each guitar an average is taken of the centroid resulting from several impulse responses, namely, knocking and measuring at the treble side of the bridge, knocking and measuring at the bass side, knocking at the middle of the bridge while measuring at both ends of the bridge. For the sound pressure the average is taken from one measure at the treble side and one measure at the bass side, both averaged between 50 Hz and 4000 Hz, and normalized to one Hz.

Again, the guitars from the different epochs clearly fall into well separable classes. The ranking is romantic, Torres, early romantic guitars, and finally contemporary guitar, in the map sound pressure versus spectral centroid. There is a notable gap between old and contemporary guitars. There are only two exceptions from a clear classification, T89 revealing a very high sound pressure, and one of the vintage flamenco guitars from Granada revealing an unusual low spectral centroid. From these observations one can conclude that modern guitars reveal significantly more brilliance than old guitars, and that modern guitars develop a higher sound pressure, normalized to input force.

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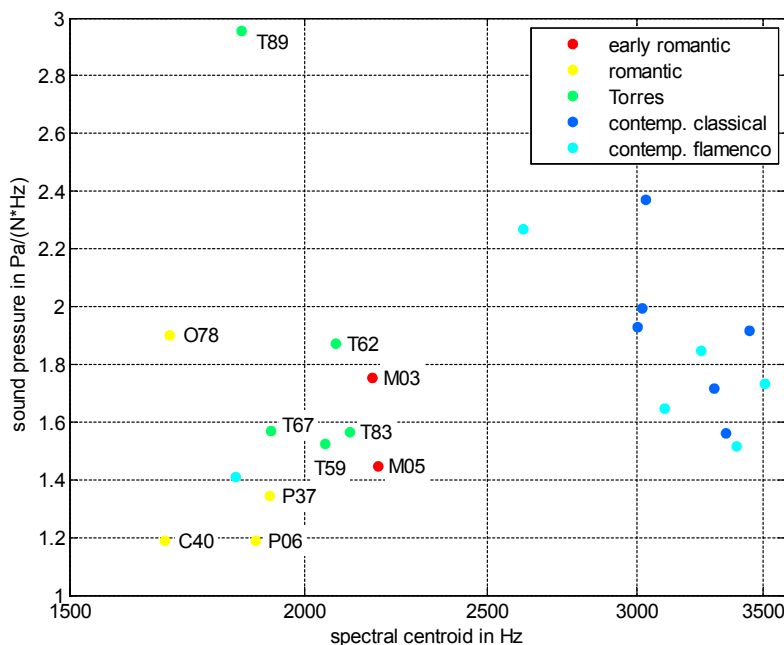


Fig. 12. Sound pressure normalized to impact force and to bandwidth in the range of 50 Hz to 4000 Hz versus spectral centroid at a sample rate of 12 kHz for guitars of various epochs. For acronyms see Table 1, for non-labelled examples of guitars see text.

There seem to be different concepts of intended timbre, or sound for each individual epoch as the classes are well separable. Interestingly, Torres' guitars fall in between the romantic guitars and the early romantic guitars. This, however, can be explained by the underrepresented bass in radiation, as explained above, which will directly shift the centroid to higher frequencies. This same argument does not apply to contemporary guitars, because the radiation at low frequencies is well developed, as can be seen in Fig. 10. In fact, the brilliance region above 1 kHz is stronger developed in contemporary guitars as well, weighing strong in centroid analysis.

SUMMARY

Torres' guitars are well distinguishable from both, romantic guitars and modern guitars likewise. Not only the shape but also the principle of sound tuning differs between the three categories. Torres increased the plantilla size to support the bass fundament around 80 to 90 Hz. He achieved this together with very soft top plates. The modern guitars do support this frequency range as well, but with an even larger plantilla combined with harder top plates. Stiffer plates radiate more efficiently, in particular at low frequencies. From this

perspective, Torres overreached for low frequencies at the expense of poor projection, and the modern guitars are a reasonable optimization. Torres work is throughout experimental. This can be read from the tuning principles followed in various epochs. There was a clear concept of how a romantic guitar should sound, and the investigated guitars strictly follow one trend line. This means, that there is little variation across different guitar makers across Andalusia and beyond. The same is true for modern guitars, both flamenco and classical. But it is different for Torres' guitars. And there is less variation across different guitar makers within the romantic or within the modern epoch than there is for just one guitar maker alone, Torres, in his experimental workshop.

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